SPIE Newsroom

10.1117/2.1200711.0908

# Freeform driver-side mirrors without blind spots

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*Controlling a single bundle of light rays enables design of a driver-side mirror that provides a wide field of view with minimal distortion.*

As most drivers know, blind spots are a dangerous problem. The crux of the issue is that flat mirrors do not provide a wide enough field of view. A solution employed by trucks and buses is to use spherical mirrors, but these produce considerable distortion. Thus, the challenge is to find shapes that yield a wide field of view without distorting the image. Here we will consider only the driver-side mirror.

The basic geometry for an automobile is depicted in Figure 1. The essential difference between these two problems is that the angle of deflection of the optical axis, $\theta$, is 90 degrees in the passenger side case, whereas for the driver side the corresponding angle, $\psi$, is closer to 60 degrees, but it can vary greatly with the position of the driver. In the United States, the law requires that the driver-side mirror be flat. (The passenger-side mirror, however, may be curved.)

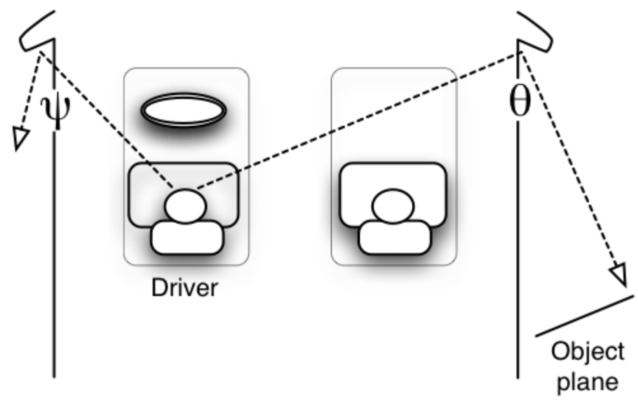


***Figure 1.*** *The geometry of the blind-spot problem.*

**Modeling the problem**

Assume we are given an optical ray bundle, a target surface, $S$, and a transformation, $T$, that assigns to each ray in the bundle a point on $S$, as shown in Figure 2. We imagine the ray bundle emanating from the driver's eye, off the driver-side mirror, and onto a plane behind the car. Having no distortion means that $T$ is scaling by one plane onto the other. The larger the scale factor, the larger the field of view of the mirror will be. The goal is to design a single reflective or refractive surface, $M$, that will take each ray to its prescribed target point. In general this is not possible, but an approximate solution to the problem may be acceptable.[1] The chances are that the surface will not have rotational symmetry, that is, it will be a freeform surface.

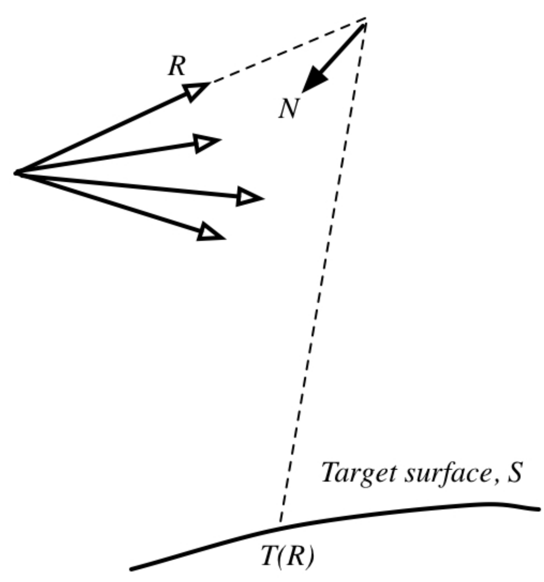


***Figure 2.*** *The question we address is how to design a surface that will take the rays from a given bundle R to prescribed target points—T(R)—on a given surface, S. This naturally gives rise to a vector field,* **N**, *which bisects the angle of the ray path as it reflects off the mirror surface.*

This construction gives rise to a vector, **N**, which is defined at every point of the bundle and will be normal to the surface, $M$, if $M$ exists. The construction of **N** is straightforward: one fixes a point on the ray, computes the unit direction back along the ray, and adds it to the unit direction from the point on the ray to the

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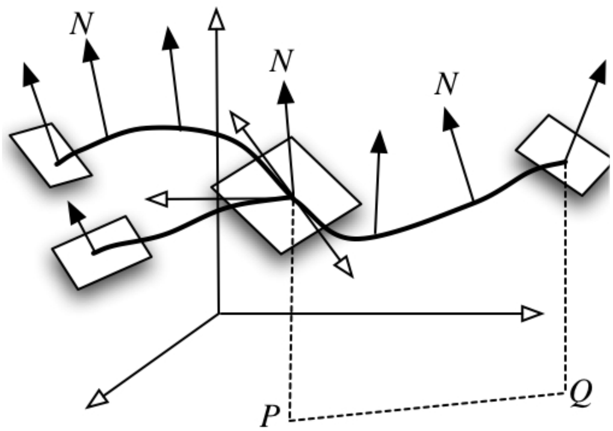


*Figure 3.* *Picking an initial height above a point P in the horizontal plane, one wishes to compute the height of a solution surface above a point Q. This determines a vertical plane. Intersection with the planes of the distribution gives a slope field, that is, essentially a differential equation, in the vertical plane. Integrating from the initial point over P will give a height over Q.*

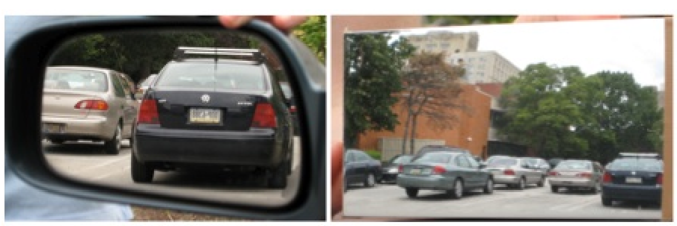

*Figure 4.* *On the left we see a parking lot via a conventional flat driver-side mirror. On the right is a view of the parking lot with the mirror designed by the author.*

target point. This gives a vector at each point of the ray bundle. It is a necessary condition that for a surface to exist that is normal to **N**, $\mathbf{N} \cdot (\nabla \times \mathbf{N}) = 0$,[1,2] but this is not the case here. Many ways of finding approximate solutions have been investigated for related problems.[3]

Note that the length of **N** is irrelevant. We really only care that the planes tangent to $M$ coincide with the orthogonal complement of **N**. In other words, we see that our problem naturally gives rise to a planar distribution, that is, an assignment of a plane to each point in the region of interest. Moreover, we wish $M$ to be an integral surface of that distribution, meaning that the tangent planes of $M$ coincide with the planes of the distribution.

Even if there is no surface normal to the vector field **N**, one can be computed. Given a proposed vector field, normal to the surface, points on the surface can be found by integrating along 'slices' of the distribution determined by **N** (see Figure 3). This is very similar to the method of characteristics,[4] which is useful for solving first-order partial differential equations. If we apply this problem to the design of a driver-side mirror, we may construct an approximate solution with a 45-degree field of view. A flat mirror provides less than a 20-degree field of view. Figure 4 provides a comparison.

**Conclusion**

We have presented a means of solving optical design problems that involve only a single ray bundle. Although our example design is a mirror, the principle also extends to lenses. One must, however, beware of total internal reflection when using this method. Thus, applications such as mirrors to be used with the human eye are appropriate. The problem of controlling many ray bundles is of course much harder, such as in the design of photographic lenses. Design methods for problems based on partial or ordinary differential equations are scarce, and for this reason, they constitute a current topic of my research.

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